

# Contents

Acknowledgments	x i
Introduction	x i i i
Special Notations	x i x
<b>I Martingales</b>	<b>1</b>
1. Histories and Stopping Times	1
2. Martingales	3
3. Predictability	8
4. Square-Integrable Martingales	11
References	12
Solutions to Exercises, Chapter I	13
<b>II Point Processes, Queues, and Intensities</b>	<b>18</b>
1. Counting Processes and Queues	18
2. Watanabe's Characterization	23
3. Stochastic Intensity, General Case	27
4. Predictable Intensities	30
5. Representation of Queues	35
6. Random Changes of Time	40
7. Cryptographic Point Processes	43
References	47
Solutions to Exercises, Chapter II	48
<b>III Integral Representation of Point-Process Martingales</b>	<b>56</b>
1. The Structure of Internal Histories	56
2. Regenerative Form of the Intensity	59
3. The Representation Theorem	64
4. Hilbert-Space Theory of Poissonian Martingales	70
5. Useful Extensions	75
References	76
Solutions to Exercises, Chapter III	77
viii	Contents
<b>IV Filtering</b>	<b>83</b>
1. The Theory of Innovations	83
2. State Estimates for Queues and Markov Chains	100
3. Continuous States and Nontrivial Prehistory	107
References	115
Solutions to Exercises, Chapter IV	115
<b>V Flows in Markovian Networks of Queues</b>	<b>122</b>
1. Single Station: The Historical Results and the Filtering Method	122
2. Jackson's Networks	131
3. Burke's Output Theorem for Networks	138
4. Cascades and Loops in Jackson's Networks	143
5. Independence and Poissonian Flows in Markov Chains	151
References	154
Solutions to Exercises, Chapter V	155
<b>VI Likelihood Ratios</b>	<b>158</b>
1. Radon-Nikodym Derivatives and Tests of Hypotheses	158
2. Changes of Intensities "à la Girsanov"	165
3. Filtering by the Method of the Probability of Reference	170
4. Applications	174
5. The Capacity of a Point-Process Channel	180
6. Detection Formula	187
References	189
Solutions to Exercises, Chapter VI	190
<b>VII Optimal Control</b>	<b>196</b>
1. Modeling Intensity Controls	196
2. Dynamic Programming for Intensity Controls: Complete-Observation Case	202
3. Input Regulation. A Case Study in Impulsive Control	211
4. Attraction Controls	219
5. Existence via Likelihood Ratio	225
References	229
Solutions to Exercises, Chapter VII	230
<b>VIII Marked Point Processes</b>	<b>233</b>
1. Counting Measure and Intensity Kernels	233
2. Martingale Representation and Filtering	238
3. Radon-Nikodym Derivatives	241
4. Towards a General Theory of Intensity	244
References	250
Solutions to Exercises, Chapter VIII	250
Contents	ix
<b>A1 Background in Probability and Stochastic Processes</b>	<b>255</b>
1. Introduction	255
2. Monotone Class Theorem	256
3. Random Variables	261
4. Expectations	266
5. Conditioning and Independence	275
6. Convergence	285
7. Stochastic Processes	287
8. Markov Processes	290
References	295
<b>A2 Stopping Times and Point-Process Histories</b>	<b>296</b>
1. Stopping Times	296
2. Changes of Time and Meyer-Dellacherie's Integration Formula	300
3. Point-Process Histories	303
References	311
<b>A3 Wiener-Driven Dynamical Systems</b>	<b>312</b>
1. Ito's Stochastic Integral	312
2. Square-Integrable Brownian Martingales	321
3. Girsanov's Theorem	327
References	332
<b>A4 Stieltjes-Lebesgue Calculus</b>	<b>334</b>
1. The Stieltjes-Lebesgue Integral	334
2. The Product and Exponential Formulas	336
References	339
<b>General Bibliography</b>	<b>341</b>